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Procedia - Social and Behavioral Sciences 96 (2013) 2042 – 2050

Procedia
Social and Behavioral Sciences

13th COTA International Conference of Transportation Professionals (CICTP 2013)

Decision-making method of highway network planning based on prospect theory

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Abstract

Considering the impact of risk attitude and the presence of grey characters for highway network planning, a grey relational optimization method based on the cumulative prospect theory is proposed. Firstly, the positive and negative ideal programs are taken as reference points through the standardization of primitive decision information. Secondly, the positive and negative grey relevancy matrices are established for taking positive and negative ideal solutions as reference sequences and taking planning programs as compared sequences. Thirdly, the positive and negative prospect value matrices and a nonlinear planning model for the maximization of comprehensive prospect value are built based on the cumulative prospect theory. The optimum weight vector is then solved, and the order of the programs is determined. Finally, the Xi'an highway network-planning program is presented as an example, which shows feasibility and effectiveness of the method.

Keywords :traffic engineering; highway network; prospect theory; grey relevancy; optimization

1. Introduction

Highway network planning evaluation is a multi-level and multi-purpose complex system. Its influence range is so extremely wide that planning evaluation should be quantitatively and qualitatively measured from technical, economic, social and environmental aspects to determine the relative value of different schemes，which can provide the scientific basis for the comparison and optimization of different highway network planning schemes (Tao et. al., 2002). At present, domestic and foreign scholars have conducted a lot of studies (Li, 2006), which puts forward the effective method of principal component analysis, grey relational analysis (Wei et. al., 1999),

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doi:10.1016/j.sbspro.2013.08.230

projection method (Li, 2005), two level fuzzy rating method (Hong & Yingqiu, 1999), and greatly promote the scientific decision making of highway network planning (Zhong, 2007).

Prospect theory was put forward by Kahneman (Kahneman & Tversky, 1979), which can't be realized by rational decision making behavior. Kahneman and Tversky think that the individual is assigned a non-probabilistic weight for different results by taking the probability into decision weighting function, which is one of the most-cited research papers in the field of economics. Subsequently, they put forward the cumulative prospect (Tversky & Kahneman, 1992), which introduces capacity probability to solve the problem of strong dominant and treatment of multiple outcomes. In recent years, prospect theory is realized and researched by domestic scholars (Wei etc., 2005) in the field of transportation on the path choice behavior (Jianqiang et. al., 2009). For example, Zhao Lin (2007) studied the traveler's route choice behavior model under the condition of finite discrete distributions based on prospect theory, Xu Hongli (2007) put forward the path choice behavior analysis method through traveler route choice survey data based on prospect theory. Yang Zhiyong (2007) studied route choice problem under the basic frame of real-time traffic information based on prospect theory.

In the real selection process of highway network planning schemes, decision makers often consider policy, city space development and many other factors, and existing subjective risk preference for planning schemes, which will directly affect the final decision effect. Therefore, it is necessary for decision-makers to consider risk factors in the decision making process of highway network planning scheme. However, the traditional decision-making methods assume that the decision makers are absolutely rational, and have complete information and the same preferences, ignoring the risk attitude of decision makers for network planning scheme decision-making process. In view of above reasons, this paper combines cumulative prospect theory with grey relational analysis, builds up grey relational optimization model considering decision-makers risk attitude on the base of cumulative prospect theory, which is consistent with the human thinking mode. In the end, the paper takes Xi'an network planning schemes as an example and illustrates that the optimization method is scientific and effective.

2. The Mathematical Theory of Decision Making

Prospect theory analyses the problem from the perspective of gains and losses, which think that people's attitude towards treating the gains and losses is asymmetric. When people face gain, they tend to be "risk averse"; when people face loss, they tend to be "risk seeking ". Evaluation based on the gains and losses is the reference point. The risk preference of people under the condition of uncertainty is a nonlinear relation, which is consistent with highway network planning evaluation principles. Correlation analysis (Lin, 2007) is a method that describes similar degree among different factors in the system, which is wildly applied in Grey Theory. The city highway network planning system is a complex gray system including multiple factors and levels, and the optimization goal in system are not independent of each other, so the gray correlation analysis method is applied to judge the schemes and reference point correlation degree.

2.1. Standardized decision matrix

Suppose there are n decision-making schemes to be selected in highway network planning process, which can be noted as $A=\{A_1, A_2, \cdots, \mathrm{A_n}\}$. Decision-making indexes were taken as index set, which can be noted as $G=\{G_1, G_2, \cdots, G_m\}$. $N=\{1,2,3\cdots, n\}$, $M=\{1,2,3,\cdots,m\}$, $i \in N$, $j \in M$.

Decision-making index weight can be noted as $w=(w_1, w_2, \cdots, w_m)$. y_{ij} shows the attribute value of G_j in the planning scheme A_i.

$Y=(y_{ij})_{n\times m}$ can be noted as follows,which shows the decision-making matrix.

$$Y = \left(y_{ij}\right)_{n\times m} = \begin{bmatrix} y_{11} & y_{12} & \cdots & y_{1m} \\ y_{21} & y_{22} & \cdots & y_{2m} \\ \cdots & \cdots & \cdots & \cdots \\ y_{n1} & y_{n2} & \cdots & y_{nm} \end{bmatrix} \tag{1}$$

2.2. Determining the ideal solution

Because decision-making indexes have different meaning and dimension, it is necessary to standardize the decision-making indexes.

For cost index G_j and y_{ij} can be standardized as follows.

$$r_{ij} = \frac{\max(y_j) - y_{ij}}{\max(y_j) - \min(y_j)} \tag{3}$$

For beneficial indexe G_j and y_{ij} can be standardized as follows.

$$r_{ij} = \frac{y_{ij} - \min(y_j)}{\max(y_j) - \min(y_j)} \tag{4}$$

In the above formula,

$$\max(y_j) = \max\left\{y_{ij}/1 < i < n\right\},\ \min(y_j) = \min\left\{y_{ij}/1 < i < n\right\}.$$

So the decision-making matrix Y after standardized manipulation can be denoted as following matrix.

$$R = \left(r_{ij}\right)_{nm} = \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1m} \\ r_{21} & r_{22} & \cdots & r_{2m} \\ \cdots & \cdots & \cdots & \cdots \\ r_{n1} & r_{n2} & \cdots & r_{nm} \end{bmatrix} \tag{5}$$

When decision-makers make a decision, they often measure the gains and losses of decision-making according to reference points based on prospect theory. The positive and negative ideal solutions were taken as reference points in view of TOPSIS.

If $r_j^+ = \max\left\{r_{ij} \middle| 1 \le i \le n\right\}, r_j^- = \min\left\{r_{ij} \middle| 1 \le i \le n\right\}$, then the positive ideal solution can be noted as follows.

$$A^+ = \left(r_1^+, r_2^+, \cdots, r_m^+\right) \tag{6}$$

The negative ideal solution can be noted as follows.

$$A^- = \left(r_1^-, r_2^-, \cdots, r_m^-\right) \tag{7}$$

2.3. Determining the positive and negative prospects matrix

Based on the grey relational analysis method (LI Xiaowei etc., 2012), the ideal solution A^+ and the negative ideal solution A^- can be taken as the reference sequences, the scheme A_i can be taken as compared sequences.
The correlation coefficient of the positive ideal scheme about optimization goals can be expressed as follows.

$$\xi_{ij}^+ = \frac{\min_i \min_j \left| r_{ij} - r_j^+ \right| + \rho \max_i \max_j \left| r_{ij} - r_j^+ \right|}{\left| r_{ij} - r_j^+ \right| + \rho \max_i \max_j \left| r_{ij} - r_j^+ \right|} \tag{8}$$

The correlation coefficient of the negative ideal scheme about optimization goals can be expressed as follows.

$$\xi_{ij}^- = \frac{\min_i \min_j \left| r_{ij} - r_j^- \right| + \rho \max_i \max_j \left| r_{ij} - r_j^- \right|}{\left| r_{ij} - r_j^- \right| + \rho \max_i \max_j \left| r_{ij} - r_j^- \right|} \tag{9}$$

In above formula, the distinguishing coefficient $\rho \in [0,1]$, and its value usually can be made by $\rho = 0.5$.
Therefore, the positive and negative correlation coefficient matrix can be expressed as follows.

$$\xi^+ = \begin{bmatrix} \xi_{11}^+ & \xi_{12}^+ & \cdots & \xi_{1m}^+ \\ \xi_{21}^+ & \xi_{22}^+ & \cdots & \xi_{2m}^+ \\ \cdots & \cdots & \cdots & \cdots \\ \xi_{n1}^+ & \xi_{n2}^+ & \cdots & \xi_{nm}^+ \end{bmatrix} \tag{10}$$

$$\xi^- = \begin{bmatrix} \xi_{11}^- & \xi_{12}^- & \cdots & \xi_{1m}^- \\ \xi_{21}^- & \xi_{22}^- & \cdots & \xi_{2m}^- \\ \cdots & \cdots & \cdots & \cdots \\ \xi_{n1}^- & \xi_{n2}^- & \cdots & \xi_{nm}^- \end{bmatrix} \tag{11}$$

2.4. Constructing the prospect value matrix

Value function is used to reflect the relationship between the expected results and subjective utility of the decision maker, which is used instead of the utility function in cumulative prospect theory. Decision-makers subjective feeling forms the value function. It has three important characteristics. Firstly, Gains and losses are relative to the reference point. Secondly, decision makers avoid risk facing earnings，decision makers prefer risk facing losses. Thirdly，losses are more sensitive than gains for decision makers (Jianqiang, 2002).

According to Wang Zhengxin (2010) research, the positive prospect value of scheme can be quantified as $v_{ij}^+ = \left(1 - \xi_{ij}^-\right)^{0.88}$, and the negative prospect value of scheme can be quantified as $v_{ij}^- = -2.25\left[-(\xi_{ij}^+ - 1)\right]^{0.88}$.

Therefore, the positive prospect value matrix can be expressed as follows.

$$V^{+}=\begin{bmatrix} v_{11}^{+} & v_{12}^{+} & \cdots & v_{1m}^{+} \\ v_{21}^{+} & v_{22}^{+} & \cdots & v_{2m}^{+} \\ \cdots & \cdots & \cdots & \cdots \\ v_{n1}^{+} & v_{n2}^{+} & \cdots & v_{nm}^{+} \end{bmatrix}$$

(12)

And, the negative prospect value matrix can be expressed as follows.

$$V^{-}=\begin{bmatrix} v_{11}^{-} & v_{12}^{-} & \cdots & v_{1m}^{-} \\ v_{21}^{-} & v_{22}^{-} & \cdots & v_{2m}^{-} \\ \cdots & \cdots & \cdots & \cdots \\ v_{n1}^{-} & v_{n2}^{-} & \cdots & v_{nm}^{-} \end{bmatrix}$$

(13)

2.5. Constructing comprehensive prospect model and solution

The bigger the comprehensive prospect value, the better for each scheme (Zhiyong & Guiyun, 2008). If prospect weight can be noted as $\pi^{+}\left(w_j\right)$ when decision makers face earnings, and prospect weight can be noted as $\pi^{-}\left(w_j\right)$ when decision makers face loss, comprehensive prospect value V_i for each scheme can be expressed as follows.

$$V_i=\sum_{j=1}^{m} v_{ij}^{+}\pi^{+}\left(w_j\right)+\sum_{j=1}^{m} v_{ij}^{-}\pi^{-}\left(w_j\right).$$

In the above formula,

$$\pi^{+}\left(w_j\right)=\frac{w_j^{\gamma^{+}}}{\left[w_j^{\gamma^{+}}+\left(1-w_j\right)^{\gamma^{+}}\right]^{1/\gamma^{+}}},\ \pi^{-}\left(w_j\right)=\frac{w_j^{\gamma^{-}}}{\left[w_j^{\gamma^{-}}+\left(1-w_j\right)^{\gamma^{-}}\right]^{1/\gamma^{-}}},$$

$\gamma^{+}=0.61$, $\gamma^{-}=0.69$.

So optimization model can be established, and objective function can be expressed as

$\max V=\left(V_1,V_2,V_3,\cdots,V_n\right)$.

Because each scheme are fair competed, optimization model M_1 can be expressed as follows.

$$\max V=\sum_{i=1}^{n}\sum_{j=1}^{m} v_{ij}^{+}\pi^{+}\left(w_j\right)+\sum_{i=1}^{n}\sum_{j=1}^{m} v_{ij}^{-}\pi^{-}\left(w_j\right)\ s.t.\sum_{j=1}^{m} w_j=1, w_j\ge 0, w\subset H.$$

Optimal solutions $w^{*}=\left(w_1^{*}, w_2^{*}, w_3^{*}, w_4^{*}, w_5^{*}\right)$ can be obtained to solve the above model, and the optimal comprehensive prospect value V_i^{*} for schemes can be expressed as follows.

$$V_i^{*}=\sum_{j=1}^{m} v_{ij}^{+}\pi^{+}\left(w_j^{*}\right)+\sum_{j=1}^{m} v_{ij}^{-}\pi^{-}\left(w_j^{*}\right).$$

Therefore, the sequence of schemes can be determined by comprehensive prospect value.

3. Empirical research

Xi'an highway network planning draws up the five kinds of planning schemes. Highway network class G_1 、Highway network saturation G_2 、 highway network travel time G_3 、 transport cost G_4 、 Accident rate G_5 are selected as decision index. The decision index information of decision makers is expressed as follows.

$$H:\left\{\begin{array}{l}0.25 \le w_1 \le 0.4, 0.15 \le w_2 \le 0.35,\\ 0.1 \le w_3 \le 0.4, 0.15 \le w_4 \le 0.20\\ 0.12 \le w_5 \le 0.20\end{array}\right\}$$

Decision index calculation values for five kinds of schemes are shown in table 1, then according to the method of optimal decision making, to verify its validity and practicability.

Table 1.The calculated values of the decision indexes about planning programs

	G_1	G_2	G_3	G_4	G_5
A_1	3.42	1.45	14132.67	290.19	1.40
A_2	2.85	1.30	13590.79	287.90	1.52
A_3	3.24	1.56	13814.17	294.76	1.30
A_4	2.93	1.24	13866.85	288.99	1.37
A_5	3.16	1.49	14150.33	290.60	1.46

Based on the above formulas (1-13), the calculation steps can be expressed as follows.

3.1. Construction of decision matrix

$$Y=\begin{bmatrix}3.42 & 1.45 & 14132.67 & 290.19 & 1.40\\ 2.85 & 1.30 & 13590.79 & 287.90 & 1.52\\ 3.24 & 1.56 & 13814.17 & 294.76 & 1.30\\ 2.93 & 1.24 & 13866.85 & 288.99 & 1.37\\ 3.16 & 1.49 & 14150.33 & 290.60 & 1.46\end{bmatrix}$$

3.2. The standard decision matrix

$$R=\begin{bmatrix}1 & -0.25 & -0.69 & 0.07 & 0.09\\ -0.9 & 0.64 & 1 & 0.61 & -1\\ 0.4 & -0.90 & 0.3 & -1 & 1\\ -0.63 & 1 & 0.14 & 0.35 & 0.36\\ 0.13 & -0.49 & -0.75 & -0.03 & -0.45\end{bmatrix}$$

3.3. Determining the positive and negative ideal scheme

$$A^{+}=(1,1,1,0.61,1),\quad A^{-}=(-0.9,-0.9,-0.75,-1,-1)$$

3.4. Construction of positive and negative correlation coefficient matrix

$$\xi^{+}=\begin{bmatrix}1 & 0.4 & 0.34 & 0.6 & 0.52\\ 0.31 & 0.70 & 1 & 0.99 & 0.33\\ 0.59 & 0.31 & 0.56 & 0.33 & 1\\ 0.34 & 1 & 0.5 & 0.76 & 0.61\\ 0.5 & 0.36 & 0.33 & 0.56 & 0.41\end{bmatrix}$$

$$\xi^{-}=\begin{bmatrix}0.31 & 0.57 & 0.94 & 0.43 & 0.48\\ 1 & 0.36 & 0.33 & 0.33 & 1\\ 1.40 & 1 & 0.45 & 1 & 0.33\\ 0.76 & 0.31 & 0.50 & 0.37 & 0.42\\ 0.45 & 0.67 & 1 & 0.45 & 0.65\end{bmatrix}$$

3.5. Determining the positive and negative outlook value matrix

$$V^{+}=\begin{bmatrix}0.72 & 0.48 & 0.09 & 0.61 & 0.56\\ 0 & 0.68 & 0.7 & 0.7 & 0\\ 0.64 & 0 & 0.59 & 0 & 0.7\\ 0.28 & 0.72 & 0.55 & 0.66 & 0.62\\ 0.59 & 0.37 & 0.01 & 0.59 & 0.4\end{bmatrix}$$

$$V^{-}=\begin{bmatrix}0 & -1.43 & -1.56 & -1.01 & -1.17\\ -1.63 & -0.77 & 0 & -0.02 & -1.57\\ -1.04 & -1.63 & -1.10 & -1.57 & 0\\ -1.56 & 0 & -1.21 & -0.65 & -0.98\\ -1.23 & -1.51 & -1.57 & -1.10 & -1.42\end{bmatrix}$$

3.6. Modeling and solving

The optimal model is established with the comprehensive prospect value maximization as the goal.

$$\max V=\sum_{i=1}^{5}\sum_{j=1}^{5}v_{ij}^{+}\pi^{+}\left(w_j\right)+\sum_{i=1}^{5}\sum_{j=1}^{5}v_{ij}^{-}\pi^{-}\left(w_j\right)$$

$$s.t.\begin{cases}0.25 \le w_1 \le 0.4, 0.15 \le w_2 \le 0.35,\\ 0.1 \le w_3 \le 0.3, 0.15 \le w_4 \le 0.20,\\ 0.12 \le w_5 \le 0.20, \sum_{j}^{5} w_j = 1\end{cases}$$

The optimal solution can be obtained by solving the model.

$$w^* = (0.25, 0.22, 0.20, 0.15, 0.18)$$

Calculating the prospect value of five schemes by put the optimal weight into the formula V_i^* .

$$V_1^* = -0.66; V_2^* = -0.45; V_3^* = -0.90; V_4^* = -0.37; V_5^* = -1.21.$$

According to the order from big to small, we can get the optimal scheme ranking.

$$A_4 > A_2 > A_1 > A_3 > A_5$$

Therefore，Scheme 4 is the best scheme.

4. Conclusion

Prospect theory is more suitable for the decision maker's actual decision-making behavior under the uncertainty cases, which provides a more effective tool for the analysis of transit network optimization decision-making problems. On the basis of previous studies, this paper gives the transit network optimization objective function, and determines the constraint conditions, and builds the comprehensive prospect value maximization model based on the cumulative prospect theory and grey system theory. Finally, Xi'an highway network planning is taken as an example to indicate effectiveness and practicality of the method. Due to the emphasis of lines optimization is different in the different city, it is necessary to adjust parameter properly in practicality to scientifically describe the influence of the decision-making behavior.

Acknowledgment

The paper has been supported by the youth foundation of Xi'an University of Architecture and Technology (No DB01138), and key discipline science incubation program of Xi'an University of Architecture and Technology (No xk201101).

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